Statistics 19 – Inverse Normal Distribution

Please **complete** this homework by ___________. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics.
Please complete all questions.

1. A set of continuous data, X, has a lower quartile, median and upper quartile to be Q_1, Q_2 and Q_3respectively.

a) Find $P(X < Q_1)$
b) Find $P(X > Q_2)$
c) Four independent values of X are taken. Find the probability that:
   i. All are below Q_3
   ii. At least two are between Q_1 and Q_2

2. $X \sim N(70, 8^2)$

Find:

a) $P(X < 60)$
b) $P(X > 80)$
c) $P(X = 55)$
d) $P(X < 62 \cup X > 78)$

3. The daily mean pressure (hPa) at Heathrow between 1st September and 31st October 2015, X, has a median 1018.5, mean 1018 and standard deviation 10.027. The number of observations between 1008 and 1028 inclusive is 42

a) Give two reasons why X is likely to be approximately Normally distributed
b) Find the probability that a randomly chosen day at Heathrow between 1st September and 31st October 2015 had a daily mean pressure between 1015 and 1025 inclusive

4. Michelle has a job repairing phones. She found that 1 in 10 phones brought in for repair had cracked screens. She suspects that over time this proportion has reduced. She carries out a hypothesis test at the 10% significance level on the next 40 phones that are brought in. She initially thought that n of these phones had cracked sreens and concluded that she should reject H_0. She then found that one more screen was cracked and concluded that she should accept $H_{0.}$ Find the value of n

5. $X \sim N(\mu, \sigma^2)$.

a) In each case below, sketch a diagram to assess whether $a < \mu$ or $a > \mu$
   i. $P(X < a) > 0.5$
   ii. $P(X < a) < 0.5$
   iii. $P(X > a) > 0.5$
   iv. $P(X > a) < 0.5$

b) What is true about $P(X < a)$ or $P(X > a)$ if $a = \mu$

Section 2 – Consolidation of this week's topic.
Please complete all questions.

1. Asma records the masses of a random sample of 100 plums from her garden in the table below.

Mass, m grams	$25 \leq m < 35$	$35 \leq m < 45$	$45 \leq m < 55$	$55 \leq m < 65$	$65 \leq m < 75$
Number of plums	3	29	36	30	2

a) Explain why the normal distribution might be a reasonable model for this distribution (2 marks)

Asma models the distribution of masses by $N(47.5, 10^2)$

b) Find the number of plums in the sample that this model would predict to have masses in the range:
   i. $35 \leq m < 45$ (2 marks)
   ii. $m < 25$ (2 marks)

c) Use your answers from b) to comment on the suitability of this model (1 mark)

d) Asma wants to use this model to predict the distribution of masses of next year's crop of plums. Comment on this. (1 mark)

2. $X \sim N(40, 10^2)$. Find the value of a for the following situations:

a) $P(X < a) = 0.2$ (1 mark)
b) $P(X > a) = 0.7$ (2 marks)
c) $P(\mu < X < a) = 0.1$ (2 marks)
d) $P(a < X < 38) = 0.25$ (3 marks)

3. $X \sim N(50,25)$.

Find the:

a) Median (1 mark)
b) Mode (1 mark)
c) Lower Quartile (2 marks)
d) Upper Quartile (2 marks)

4. A packing plant fills bags with cement. The weight X kg of a bag of cement can be modelled by a normal distribution with mean 49 kg and standard deviation 3 kg.

a) Find P(X > 55) (1 mark)

b) Find the weight that is exceeded by 92% of the bags (2 marks)

c) Five bags are selected at random. Let Y represent the number of bags that weigh above 55kg. Find:

   i. the distribution of Y (1 mark)

   ii. the probability that exactly one bag weighs more than 55 kg (1 mark)

   iii. the probability that all bags weigh less than 55kg (1 mark)

5. The weight of a particular variety of orange is normally distributed with mean 205g and standard deviation 25g.

a) Determine the probability that the weight of an orange is

   i. Less than 250g (1 mark)

   ii. Between 200g and 250g (1 mark)

b) Charles, a wholesaler decides to grade such oranges by weight. He decides that the smallest 30% should be graded as small, the largest 20% should be graded as large, and the remainder graded as medium. Determine to one decimal place the, the maximum weight of an orange graded as:

   i. Small (1 mark)

   ii. Medium (2 marks)

c) Charles claims he can model L, the number of large oranges selected by his customers by using a binomial distribution with $n = 5$ and $p = 0.2$.
Which of the binomial conditions are unlikely to be satisfied. (2 marks)

Total: 35 Marks